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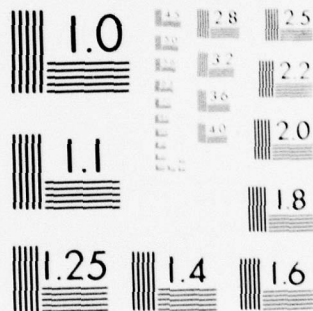
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6 "MODE AND BOUND APPROXIMATION METHODS FOR  
LARGE DEFLECTIONS OF DYNAMICALLY LOADED STRUC-  
TURES WITH PLASTIC AND VISCOPLASTIC BEHAVIOR"

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Final Technical Report

"Mode and Bound Approximation Methods for  
Large Deflections of Dynamically Loaded Structures with Plastic and Viscoplastic Behavior"

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### Objectives of Research Program

The research aimed at finding and developing methods for estimating the main features of response of engineering structures subjected to severe dynamic loading of pulse type, with emphasis put on methods valid both for large deflections and for structures of materials exhibiting strong strain rate sensitivity in the plastic range. Problem types of practical importance include explosive loading, either external due to military attack or internal for example due to disruptive accident in a pressure vessel or containment structure; various types of vehicular impact; wave impact on ship or offshore structures; and high energy rate forming. Preliminary applications have been made of the methods under investigation in the program presently being reviewed in all but the last of the above areas.

The need for reliable estimation techniques arises in part simply from the need to extend basic knowledge, and in part from the inadequacy of approaches by wholly numerical schemes. The latter have been the object of research and development on a large scale over the past decade, particularly for methods of finite element type. Despite this effort examples are familiar to most of us of cases where a supposedly tested numerical code, applied in slightly different circumstances, has led to misleading or totally nonsensical results, often after substantial computing cost. It is not surprising that this is the case. The problems concerned are intrinsically difficult when approached by numerical methods. These are necessarily of step-by-step nature, proceeding from given initial surface tractions and continuing with specified loading and constraint conditions. The initial response consists of propagation of various types of elastic wavess. Traverses of these waves lead to plastic strain waves, which



build up large deformations from a complex interplay of strain rate dependent and irreversible dissipative mechanisms. Problems of convergence and stability are a fundamental hazard in all numerical schemes, which can probably never be eliminated. The need for simple but reliable and rigorously based estimation techniques is obvious, both as an adjunct to full numerical methods, and as a means of gaining better understanding of basic response phenomena that wholly numerical methods are incapable of providing.

The estimation methods investigated under the program here reviewed were of two types: theorems giving bounds on deflections and response times, and methods in which "natural" responses in simple mode (separated variable) form are used to approximate the actual dynamic response history. These are based directly on overall energy and momentum conservation principles: the bound theorems on the minimum potential energy theorem, and the mode approximation technique on a "minimum error" device which can be identified with a statement of momentum conservation. Such approaches can be expected to guide and strengthen the analyst's intuitive feel for the essential features of the response. While the numbers obtained will not be exact or in some cases even good approximations, they will be cheap and will rarely be misleading or nonsensical, which cannot be said for wholly numerical approaches. Although the problems dealt with so far are prototype problems of fairly modest complexity, results from them generally indicate that the objectives stated are realistic, even though they have not yet been fully attained.

### Work Done Including Reports and Publications

The results obtained are described in nine reports. These are the first nine items in the list of Reports and Publications, and referred to here by number in Part A of that list. Some additional references cited are included in Part B of the same list.

The two approaches investigated referred to as Deflection Bounds and the Mode Approximation Technique, although conceptually distinct, are in fact linked to each other. However, for convenience work is summarized under the two headings.

### Deflection Bound

A new bound theorem was presented [1, 2] for a structure subjected to impulsive loading. The theorem is applicable to structures whose material exhibits strongly rate sensitive behavior in the plastic range, and applies to finite strains and finite deflections. The latter are of particular importance when the structure geometry and constraints are such that substantial qualitative changes occur in the stress field during the response and the new theorem allows consideration of these effects together with viscoplastic behavior.

The new theorem is based on the principle of minimum potential energy. We are considering a structure with specified kinematic constraints and initial velocity field that corresponds to the applied impulsive pressures. The material behavior may be of any type including time dependent inelastic as well as elastic behavior. For such a material (nonholonomic) the potential energy minimum principle is not valid, in general. It becomes valid and is available in general form for our purposes when use is made of the concepts of "extremal paths" in strain and stress space which render minimum and maximum the specific work

and specific complementary work, respectively. These concepts provide the rigorous basis of the new theorem. A static load traction system is introduced. The bound on final deflection at any point, due to the dynamic loading, is obtained in terms of the deflections reached under the defined static loads. The total strain energy in the structure due to the static loading, computed according to extremal path material behavior, must be at least as great as the initial kinetic energy corresponding to the specified initial velocity field.

In applying the theorem to several prototype problems, as in [1, 2, 8], no attempt was made to use constitutive laws expressing general time dependent behavior. (These are not well established, in any case, since the required experimental results are not available). The simplest forms appropriate for the more important strongly rate sensitive metals such as mild steel, stainless steel, and titanium, are ones of essentially viscous type, with strain rates written as functions of stress only, and vice versa. Other parameters such as plastic strain, plastic work, temperature etc. are treated as parameters. These can easily be included or changed as may be necessary for particular estimation purposes. The use of essentially viscous constitutive relations in the sense defined, implies that plastic rate sensitivity is the primary material behavior. These relations include perfectly plastic behavior as a special case. The situation is exactly analogous to the use of perfectly plastic behavior as the basis for plastic limit analysis in conventional structural theory; without this idealization the limit theorems would not exist, and this powerful tool for analysis and design would not be available.

A further step in the application of the basic theorem is the introduction of homogeneous relations of viscous type. Strain rate test data are conveniently plotted as curves of stress at fixed strain levels as function of strain rate



on a logarithmic scale. For the three structural metals mentioned, and some others, the curves are concave upward. The experimental curves then can be represented very closely by equations of the form

$$\sigma = a + b\dot{\epsilon}^{1/n} \quad \text{or} \quad \frac{\sigma}{\sigma_0} = 1 + \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right)^{1/n} \quad (1a,b)$$

where  $\sigma > 0$ ,  $\dot{\epsilon} > 0$  are a uniaxial stress and conjugate strain rate, respectively, and  $a$ ,  $b$ ,  $n$  (or equivalently  $\sigma_0$ ,  $\dot{\epsilon}_0$ ,  $n$ ) are constants for a given strain level. More convenient for many purposes are homogeneous forms. (For example, the stress path for maximum complementary work takes a particularly simple form when the stress-strain rate relations are homogeneous.) If the constants in Eqs.(1) are determined from experiments, a homogeneous relation can be derived from them in the form [10]

$$\frac{\sigma}{\sigma_0} = \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right)^{1/n'} \quad (2)$$

$$\text{where} \quad \frac{n'}{n} = v = \frac{1 + \beta^{1/n}}{\beta^{1/n}}; \quad \frac{\sigma_0'}{\sigma_0} = u = \frac{1 + \beta^{1/n}}{\beta^{1/n'}} \quad (3a,b)$$

and  $\beta = \dot{\epsilon}/\dot{\epsilon}_0$ ,  $\dot{\epsilon}$  being the strain rate at which the expressions (1) and (2) are "matched", in the sense that the two curves touch with common tangent at this point. The homogeneous form (2) always lies below the inhomogeneous form (1); hence the replacement of (1) by (2) leads to a consistent error in deflection estimate of conservative type. Comparison of solutions for one and two degree of freedom problems, and for the few continuous structures for which solutions are available, have shown extremely small errors when the matching strain rate  $\dot{\epsilon}$  is representative of the highest strain rates occurring in the response. The determination of the inhomogeneous form (1) from test data is

discussed in [2], together with comparisons between final deflections and response times from the two types of representation. Both are readily written for general combined stress states and in terms of appropriate generalized stress and conjugate strain rates. This is done in various forms in references [1-9]. Effects of strain rate history are, of course, not considered in either form.

The application of the theorem requires the solution of a problem of static loading. To obtain an upper bound on the deflection at a certain point and in a specified direction, a force at this point parallel to the stated direction is required to induce total strain energy at least as large as the initial kinetic energy of the dynamic problem. This was approached by iterative schemes in [1, 2, 6]. In [6] the equations of the bound calculation were written in forms identical to those of the equations involved in the mode iteration, with different meanings of constant parameters.

A difficulty arises because of the presence of the response time of the dynamic problem in the basic inequality of the method. This can be replaced by an upper bound on this time. Although both upper and lower bounds on response time can be calculated by known methods, these are valid under assumptions of "small deflections" and hence not strictly applicable to the present problems. However, the deflection estimate is extremely insensitive to the value of response time, which appears in the form  $t_f^{1/n'}$ , where  $n'$  for the structural metals of interest here is at least 5 and may be as large as 15 or 20.

Experiments are necessary to investigate the practical validity of computed deflection bounds; the calculation is not exact, but the signs of errors due to the idealizations and simplifications made can be assessed. Experiments were therefore made on small frames of mild steel and titanium, including

material behavior tests at moderately high rates. These experiments are described in detail in reference [7]. The tests were fairly comprehensive. For both materials two types of frame were tested, corresponding to impulsive loading, concentrated or distributed over the beam member. For each material and frame loading type, impulses were applied of magnitudes over a wide range, so that the maximum permanent deflections ranged from elastic magnitudes to about a third of the span (40 or more thicknesses).

The calculated deflection bounds were found in all cases to lie above the observed final deflections of the tests. The major idealizations and simplifications were deliberately of a consistently conservative nature (i.e. such as to increase the deflection bound). The bound magnitudes were found to exceed the test values by factors as large as 50 to 75 percent. In view of the simplicity and cheapness of the calculation compared to a full numerical solution, over-estimates of this extent would be entirely acceptable for most practical purposes.

Although the calculation required is certainly "relatively simple", in its form as illustrated in references [1, 2, 6] the basic simplicity is to some extent obscured by manipulations. Work is now proceeding (under other sponsorship) to make the calculation more direct and systematic, in order to enhance the practical utility of the method.



### Mode Approximations

The so-called "mode approximation technique" was proposed by Martin and Symonds [11] for impulsively loaded structures. The basic theory was established for rigid-perfectly plastic behavior and for small deflections. It was later shown by Lee and Martin [12, 13] and Lee [14, 15] that structures of other material behaviors, in particular viscoplastic, could be treated in the same scheme. The concept of "instantaneous mode solution" was shown to be available for viscoplastic behavior which because of the constitutive equations are nonhomogeneous, does not permit mode form solutions to persist. The procedure was limited to small deflections, and even for these was rather lengthy.

Initial contributions under the present research program [2] showed a way of applying the basic concepts of the mode approximation method to impulsively loaded structures reaching large deflections, including the important cases where the response changes qualitatively at finite deflections, and with a possibility of extension to more general pulse loading and material behavior. At the same time the handling of strongly rate sensitive plastic behavior was shortened by the use of the homogeneous "matched viscous" representation [10]. As already pointed out above, this simplification is not only realistic but consistently conservative.

The proposed new approach makes use of special "natural motions" in separated-variable (mode) form. These satisfy the full system of field equations (dynamics, compatibility, constitutive equations) but do not in general agree with specified initial velocity values due to impulsive loading. The existence of persistent ("permanent") solutions of this form requires certain conditions to be met, in particular linearity of the equations of dynamics and compatibility and homogeneity of the constitutive equations. When these conditions are not satisfied, as in large deflection problems, instantaneous mode



form solutions can still be identified for a particular deflection field and measure of velocity magnitude. The amplitude of the initial mode form solution is chosen, as in the original small-deflection version, so as to minimize the initial value  $\Delta^0$  of the functional

$$\Delta[\dot{u}_i(x,t) - \dot{u}_i^*(x,t)] = \frac{1}{2} \int_V \rho (\dot{u}_i - \dot{u}_i^*)(\dot{u}_i - \dot{u}_i^*) dV \quad (4)$$

where  $\dot{u}_i$ , ( $i = 1, 2, 3$ ) is the actual velocity field and  $\dot{u}_i^*$  is that of a mode form solution;  $\rho$  is mass density, and the integral covers the structure. The convergence property ( $d\Delta/dt \leq 0$ ) can no longer be proved for large deflections, but it may still be postulated that the "best" initial mode field remains the one which minimizes the initial mean square difference between the specified and the mode field, with mass density as a weighting function. This provides a simple and advantageous starting condition, which is generally better than others, for example matching initial kinetic energy.

The new approach was applied first to a simple discrete structural model [2] and then to a circular plate [3, 8] and rectangular frame [6]. The main problem in these applications is the determination of the mode velocity field from the field equations, together with the associated acceleration magnitude. This nonlinear eigen problem was solved by iterative schemes. In [8] this was done making use of a finite element format.

Comparisons with experiments are essential to test the validity of such an approach, which makes use of idealizations and simplifications and which involves also a definite "intrinsic" error (in determining the starting velocity amplitude by the "minimum  $\Delta^0$ " technique.) Hence tests were carried out on circular plates and frames, designed to investigate both types of error. The tests on frames [7] have already been mentioned. Those on plates were also

quite comprehensive. Two rate sensitive metals were used (mild steel and titanium), and three loading conditions, differing in the degree of concentration of the impulsive pressures. The specimens were "fully clamped" circular plates. For each of the six cases a range of total impulse magnitudes was applied so that final permanent displacements ranged from elastic magnitudes to about eight plate thicknesses. Full details of the tests are given in [9], including the determination of constants for strain rate sensitive behavior and techniques for measurement of impulse and time history of displacement. The approximate determination of response by a sequence of instantaneous modes is described in [8]. Here the calculation is somewhat improved in efficiency over the earlier version [3], by making use of "master response curves" for structures of given geometry and class of material behavior. An iterative scheme to determine instantaneous mode velocity fields was still employed, but this was cast in finite element format for convenience. Convergence difficulties were encountered at large deflections, but from empirical observations these are believed to be of no importance as far as final deflections are concerned.

The comparison of test results (final maximum deflections, deflection profiles, and response times) with predictions of the mode approximation technique are discussed in [8]. Here emphasis is put on identifying the main sources of error. The agreement between test deflection magnitudes and those predicted is in general quite good, although not so close as found in the frame tests (see [6, 7]). This agreement is less important for present purposes than understanding why good agreement should, or should not, be expected. In both the frame and the plate tests, some anomalous results remain to be explained, as discussed in [6] and [8]. Although the "clamped plate" test is a prototype of the type

of structure whose response changes qualitatively as large deflections are reached, the full clamping condition is experimentally difficult to achieve.

It should be emphasized that the "mode technique" is not merely a device for treating dynamic plastic structural response by one degree of freedom models and ad hoc short-cuts. The engineering literature is full of approaches of this nature; by far the most rational and general such approach is that of Kaliszky [16, 17]. The concepts of mode form response for structures of certain classes of inelastic behavior are as fundamental in dynamic plastic theory as are the concepts of limit load and plastic collapse in static structural analysis. Close links exist between limit analysis and dynamic plastic mode response. Some of these were suggested by Lee [14] and Martin [18]. Our research program included contributions to understanding these links, both from a direct point of view illustrated by structural examples [5], and from that of general variational-extremal properties which characterize mode form responses [4]. The latter discussion was motivated by the hope of improving practical computational schemes by the availability of well understood variational-extremal theorems of both kinematic and dynamic type.



### Summary

The research completed and described in the nine reports or publications of References 1-9, has made a contribution to the development of a branch of basic theory underlying response of structures to severe pulse or impact loading [2, 4, 5]. This theory has direct application to rigorously based general methods for obtaining estimates of the final maximum deflections and response times, as exemplified by [1, 2, 3, 6, 8]. Rather comprehensive series of experiments were carried out [7, 9] on structures of types chosen to exhibit different large deflection behaviors of particular interest. The generally close agreement between predicted and observed deflections and times was critically discussed in [6, 8] with emphasis on the sources of error.

A considerable increase in understanding of some fundamental features of dynamic plastic response, as contrasted with facility in producing sheets of computational print-outs, may be said to have been achieved.

A number of important questions remain unsolved, concerned particularly with implementing the approximation techniques in a more standard and systematic fashion. These are now being pursued under new auspices with more adequate funding.



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